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Three-dimensional diffusion in a multiphase body with randomly disposed inclusions of a spherical form

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Abstract

The three-dimensional problem of admixture diffusion is considered in a semispace of random geometrical configuration composed of $N+1$ phases under action of a constant source on the body surface. The used approach for description of admixture transfer in a random nonhomogeneous body allows for both jump discontinuities of a diffusion coefficient on interphases and equally probable distribution of spherical inclusions in the body. Admixture concentration averaged over the ensemble of phase configurations has been obtained under consideration of medium nonhomogeneities as internal sources.

1. Introduction

Nonhomogeneities of a local structure, in particular in the form of inclusions of another material, affect essentially on admixture mass transfer processes in real media. As a rule, specific disposition of such inclusions is unknown, however we know accurately enough their physical–mechanical properties, for example, density and diffusion coefficient of admixture particles, and their form (layer, cylinder, sphere, etc.).

Description of influence of single inclusions on diffusion process can be made on the basis of solutions of classical contact and initial-boundary value problems of mathematical physics [1–3]. At a large number of such nonhomogeneities, homogenization methods [4,5] are used or corresponding effective diffusion coefficients are introduced [6–8]. If we can suppose that macroscopically large quantity of particles composing inclusions, occur within a physically small element of a body then continual approaches can be applicable for mass transfer description [9–11].

At the same time some researchers [7,8] noted that application of effective diffusion coefficients describe uncompletely features of admixture mass transfer, and the conditions of satisfiability of continual models do not hold always.

Accounting the influence of stochastically disposed inclusions on mass transfer with sufficient difference of their physical characteristics (density and diffusion coefficients) from parameters of a basic material, can be done by using generalized functions, Green function for an effective medium and averaging over the ensemble of nonhomogeneity configurations [12]. Such an approach was applied for description of diffusive process in a stratified semispace where the volumetric fraction of inclusions is small [13] and commensurable the basic material [12] and also in a stratified layer [14]. In the latter paper admixture concentrations were obtained at equally probable and beta distributions of inclusions in a body (a priory information). In the paper [15] it was considered diffusion in a multiphase stratified semispace under random configuration of sublayers.

The purpose of this work is generalization of the proposed approach for studing diffusive processes in a semispace with randomly disposed spherical inclusions of distinct materials.

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2. Subject of inquiry and problem formulation

We study admixture particle transport in a dispersed isotropic semispace of multiphase random nonhomogeneous structure of material. The body is composed of $N+1$ phases with distinct densities: matrix (the basic phase, marked by the index 0) and spherical inclusions of N kinds that are disposed randomly in the body space (see Fig. 1). Admixture diffusion coefficients can differ substantially in these phases. We assume that each phase is distributed by equally probable distribution in the body region. We consider a case where a volumetric fraction v_0 of the basic phase is much greater then the others: $v_0 \gg v_j,\ j = 1, 2, \ldots, N$. If the body volume is denoted V then

$$\bigcup_{i=1}^{n_j} V_i^{(j)} = V^{(j)}, \quad \bigcup_{j=0}^{N} V^{(j)} = V, \tag{1}$$

where $V^{(j)}$ is a volume of j-phase; $V_i^{(j)}$ is a volume of an inclusion i of j-phase, i is the inclusion number, $i = 1, 2, \ldots, n_j$, n_j is a number of j-kind inclusions. And we assume that the body density $\rho(\mathbf{r})$ and admixture diffusion coefficient $D(\mathbf{r})$ are constant in the space of each phase ($\mathbf{r}$ is a radius-vector of a current point).

Let us introduce into consideration the random operator $\eta_{ij}(\mathbf{r})$ that depends on the phase configuration and does not depend on their physical characteristics. It is defined such as

$$\eta_{ij}(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in V_i^{(j)}, \\ 0, & \mathbf{r} \notin V_i^{(j)}, \end{cases} \quad \sum_{j=0}^{N} \sum_{i=1}^{n_j} \eta_{ij}(\mathbf{r}) = 1. \tag{2}$$

Then the diffusion coefficient $D(\mathbf{r})$ and the body density $\rho(\mathbf{r})$ are presented by the random operator (2) as follows

$$D(\mathbf{r}) = \sum_{j=0}^{N} \sum_{i=1}^{n_j} D_j \eta_{ij}(\mathbf{r}), \quad \rho(\mathbf{r}) = \sum_{j=0}^{N} \sum_{i=1}^{n_j} \rho_j \eta_{ij}(\mathbf{r}), \tag{3}$$

where D_j, ρ_j denote values of respective coefficients in the j-phase.

Using the approach of generalized functions [3,16] admixture diffusion in a random nonhomogeneous multiphase body is described in the form:

$$L(\mathbf{r},t)c(\mathbf{r},t) \equiv \bar{\rho}(\mathbf{r}) \frac{\partial c(\mathbf{r},t)}{\partial t} - \nabla[D(\mathbf{r})\nabla c(\mathbf{r},t)] = 0. \tag{4}$$

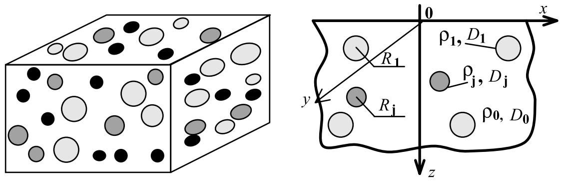

Fig. 1. One of possible realization of a body structure.

Here $c(\mathbf{r},t)$ denotes the field of admixture concentration in the body, $\bar{\rho}(\mathbf{r}) = \rho(\mathbf{r})/\rho_0$ is a random normalized density, $\nabla = \mathbf{i}(\partial/\partial x) + \mathbf{j}(\partial/\partial y) + \mathbf{k}(\partial/\partial z)$; x, y, z are the space coordinates, t is time, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors of Cartesian coordinates.

Let a constant mass source act on the boundary of the semispace referred to rectangular coordinates

$$c(\mathbf{r},t)|_{z=0} = c^* \equiv \text{const},$$

another boundary conditions and initial one are also given

$$c(\mathbf{r},t)|_{z\to\infty} = 0, \quad c(\mathbf{r},t)|_{x,y\to\pm\infty} \leqslant K < \infty, \quad c(\mathbf{r},t)|_{t=0} = 0. \tag{5}$$

Substitute coefficient representation (3) into Eq. (4) and allow for that on interphases [16]

$$\sum_{j=0}^{N} \sum_{i=1}^{n_j} \nabla\left(D_j \eta_{ij}(\mathbf{r})\right) = \sum_{j=0}^{N} \sum_{i=1}^{n_j} [D_j]_\Gamma \delta\left(\mathbf{r} - \mathbf{r}_{ij}^\Gamma\right),$$

where $[D_j]_\Gamma$ denotes a jump of the diffusion coefficient on the boundary of the inclusion $(V_i^{(j)})$, $\delta(\mathbf{r})$ is Dirac delta-function, $\mathbf{r}_{ij}^\Gamma$ is a radius-vector of points on the boundary of subregion $(V_i^{(j)})$ (random magnitude). Then we obtain

$$L(\mathbf{r},t)c(\mathbf{r},t) = \sum_{j=0}^{N} \sum_{i=1}^{n_j} L_{ij}(\mathbf{r},t)c(\mathbf{r},t) = 0, \tag{6}$$

where the random operator L_{ij} is

$$L_{ij}(\mathbf{r},t) = \bar{\rho}_j \eta_{ij}(\mathbf{r}) \frac{\partial}{\partial t} - D_j \eta_{ij}(\mathbf{r}) \nabla^2 - \left[D_j\right]_\Gamma \delta(\mathbf{r} - \mathbf{r}_{ij}^\Gamma)\nabla. \tag{7}$$

3. Neyman series for the diffusion problem

In Eq. (6) add and subtract the deterministic operator $L_0(\mathbf{r},t)$ defined at all interval ($t \in [0;\infty[, x, y \in]-\infty;\infty[, z \in [0;\infty[$) as

$$L_0(\mathbf{r},t) \equiv \bar{\rho}_0 \frac{\partial}{\partial t} - D_0\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right], \tag{8}$$

which coefficients are characteristics of the basic phase. Then taking into account the condition (2) we have

$$L_0(\mathbf{r},t)c(\mathbf{r},t) = L_s(\mathbf{r},t)c(\mathbf{r},t), \tag{9}$$

where the random operator L_s is obtained in the form

$$\begin{aligned} L_s(\mathbf{r},t) &\equiv L_0 - L \\ &= \sum_{j=1}^{N} (\bar{\rho}_0 - \bar{\rho}_j) \sum_{i=1}^{n_j} \eta_{ij}(\mathbf{r}) \frac{\partial}{\partial t} - \sum_{j=1}^{N} (D_0 - D_j) \\ &\quad \times \sum_{i=1}^{n_j} \eta_{ij}(\mathbf{r}) \nabla^2 + \sum_{j=1}^{N} [D_j]_\Gamma \sum_{i=1}^{n_j} \delta(\mathbf{r} - \mathbf{r}_{ij}^\Gamma)\nabla. \end{aligned} \tag{10}$$

We consider the right-hand side of Eq. (9) as a source, i.e. medium nonhomogeneity is treated as internal sources. The solution of the initial-boundary value problem (9) and (5) is found in the form of Neyman series [17].

Let $c_0(\mathbf{r},t)$ is a deterministic field of the admixture concentration in the body with characteristics $\bar{\rho}_0$, D_0. It satisfies the following homogeneous equation

$$L_0(\mathbf{r},t)c_0(\mathbf{r},t) \equiv \bar{\rho}_0 \frac{\partial c_0}{\partial t} - D_0\left[\frac{\partial^2 c_0}{\partial x^2} + \frac{\partial^2 c_0}{\partial y^2} + \frac{\partial^2 c_0}{\partial z^2}\right] = 0$$

and initial and boundary conditions (5). Taking into account symmetry in a variable z, we have [2]

$$c_0(\mathbf{r},t) \equiv c_0(z,t) = c^*\mathrm{erfc}\left\{\frac{\sqrt{\bar{\rho}_0}z}{2\sqrt{D_0 t}}\right\}. \tag{11}$$

Write $G(\mathbf{r},\mathbf{r}',t,t')$ for unperturbed Green function satisfying the diffusion equation for a point source

$$\bar{\rho}_0 \frac{\partial G}{\partial t} - D_0\left[\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 G}{\partial z^2}\right] = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}') \tag{12}$$

and initial and boundary conditions

$$G(\mathbf{r},\mathbf{r}',t,t')\big|_{t=0} = 0, \quad G(\mathbf{r},\mathbf{r}',t,t')\big|_{\substack{z=0\\ z\to\infty}} = G(\mathbf{r},\mathbf{r}',t,t')\big|_{x,y\to\pm\infty} = 0. \tag{13}$$

Then the initial-boundary value problem (9) and (5) is equivalent to the integro-differential equation for the random field of the admixture concentration $c(\mathbf{r},t)$ in a $N+1$ phase semispace:

$$c(\mathbf{r},t) = c_0(\mathbf{r},t) + \int_0^t\int_V G(\mathbf{r},\mathbf{r}',t,t')L_s(\mathbf{r}',t')c(\mathbf{r}',t')\,\mathrm{d}\mathbf{r}'\,\mathrm{d}t', \tag{14}$$

where Green function is

$$G(\mathbf{r},\mathbf{r}',t,t') = \frac{1}{16\bar{\rho}_0}\left(\frac{\pi\bar{\rho}_0}{D_0(t-t')}\right)^{3/2}\exp\left\{-\frac{\bar{\rho}_0}{4D_0(t-t')}\times\left[(x-x')^2+(y-y')^2\right]\right\}\times\left[\exp\left\{-\frac{\bar{\rho}_0(z-z')^2}{4D_0(t-t')}\right\} - \exp\left\{-\frac{\bar{\rho}_0(z+z')^2}{4D_0(t-t')}\right\}\right]. \tag{15}$$

Neyman series for the problem (9) and (5) is built by iterating [17] the integro-differential equation (14). Let us restrict to the first two terms in Neyman series. Then we obtain

$$c(\mathbf{r},t) \approx c(z,t) + \int_0^t\int_V G(\mathbf{r},\mathbf{r}',t,t') \times \sum_{j=1}^{N}\sum_{i=1}^{n_j}\eta_{ij}(\mathbf{r}')\left[(\bar{\rho}_0-\bar{\rho}_j)\frac{\partial c_0}{\partial t'} - (D_0-D_j)\frac{\partial^2 c_0}{\partial z'^2}\right]\mathrm{d}\mathbf{r}'\,\mathrm{d}t' + \sum_{j=1}^{N}[D_j]_\Gamma \times \int_0^t\int_V G(\mathbf{r},\mathbf{r}',t,t')\sum_{i=1}^{n_j}\delta(\mathbf{r}'-\mathbf{r}_{ij}^\Gamma)\frac{\partial c_0}{\partial z'}\,\mathrm{d}\mathbf{r}'\,\mathrm{d}t'. \tag{16}$$

Remark that the physical interpretation of the expansion into integro-differential series (16) is analogous to one presented in the paper [12].

4. Averaging approximate solution

Let us average the expression for the random field of concentration (16) over the ensemble of phase configurations. We suppose that all phases are distributed with equally probable distribution in the body. As $c_0(z,t)$ is a deterministic field, then $\langle c_0(z,t)\rangle_{\mathrm{conf}} = c_0(z,t)$. Now consider averaging two integral terms in (16). So far as configuration of inclusions disposition is unknown then random magnitudes are radius-vectors of inclusion centres or interphase boundaries. And a random function in the first integral of (16) is only $\eta_{ij}(\mathbf{r})$. So long as

$$\eta_{ij}(\mathbf{r}') = \begin{cases}1, & \mathbf{r}'\in V_i^{(j)}\\ 0, & \mathbf{r}\notin V_i^{(j)}\end{cases} = \begin{cases}1, & |\mathbf{r}'-\mathbf{r}_{ij}|\in[0;R_j]\\ 0, & |\mathbf{r}'-\mathbf{r}_{ij}|\notin[0;R_j]\end{cases} = \eta_{ij}(|\mathbf{r}'-\mathbf{r}_{ij}|), \tag{17}$$

where $\mathbf{r}_{ij}$ is a radius-vector of the centre of the inclusion $(V_i^{(j)})$, R_j denotes a characteristic (mean) radius of the j-kind inclusions, then

$$\langle I_1\rangle_{\mathrm{conf}} = \int_0^t\int_V\left\{G\sum_{j=1}^{N}\left[\rho^*\frac{\partial c_0}{\partial t'} - D_j^*\frac{\partial^2 c_0}{\partial z'^2}\right]. \times\frac{1}{V}\sum_{i=1}^{n_j}\int_V\eta_{ij}(\mathbf{r}')\,\mathrm{d}\mathbf{r}_{ij}\right\}\mathrm{d}\mathbf{r}'\,\mathrm{d}t',$$

where $\rho^* = \bar{\rho}_0 - \bar{\rho}_j$, $D_j^* = D_0 - D_j$. Taking into account (17) and the properties of function $\eta_{ij}(|\mathbf{r}'-\mathbf{r}_{ij}|)$ we can write

$$\frac{1}{V}\sum_{i=1}^{n_j}\int_V\eta_{ij}(\mathbf{r}')\,\mathrm{d}\mathbf{r}_{ij} = \begin{cases}v_j\left(\frac{z'}{R_j}-1\right)^3, & z'<2R_j,\\ v_j, & z'\geqslant 2R_j.\end{cases}$$

Then we have

$$\langle I_1\rangle_{\rm conf}=\sum_{j=1}^{N}v_j\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\int_0^{2R_j}G(\mathbf{r},\mathbf{r}',t,t')\left(\frac{z'}{R_j}-1\right)^3\right.\times\left[\rho^*\frac{\partial c_0}{\partial t'}-D_j^*\frac{\partial^2 c_0}{\partial z'^2}\right]{\rm d}z'+\int_{2R_j}^{\infty}G(\mathbf{r},\mathbf{r}',t,t')\times\left.\left[\rho^*\frac{\partial c_0}{\partial t'}-D_j^*\frac{\partial^2 c_0}{\partial z'^2}\right]{\rm d}z'\right){\rm d}x'\,{\rm d}y'\,{\rm d}t'. \tag{18}$$

Consider averaging the second integral in (16). Since function $\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})$ depends only on a form and does not depend on medium characteristics then the correlative function between $[D_j]_{\Gamma}$ and $\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})$ equals zero. Then

$$\left\langle [D_j]_{\Gamma}\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})\right\rangle_{\rm conf}=\langle [D_j]_{\Gamma}\rangle_{\rm conf}\left\langle\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})\right\rangle_{\rm conf}. \tag{19}$$

At that

$$\left\langle [D_j]_{\Gamma}\right\rangle_{\rm conf}=D_m-D_j,$$

where $D_m=\sum_{k=0}^{N}v_kD_k$. Then the averaged second integral of (16) can be written in the form

$$\langle I_2\rangle_{\rm conf}=\sum_{j=1}^{N}(D_m-D_j)\int_0^t\int_V\left(G(\mathbf{r},\mathbf{r}',t,t')\frac{\partial c_0}{\partial z'}\times\sum_{i=1}^{n_j}\frac{1}{V}\int_V\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})\,{\rm d}\mathbf{r}_{ij}^{\Gamma}\right){\rm d}\mathbf{r}'\,{\rm d}t'.$$

Allowing for the properties of Dirac delta-function we obtain

$$\sum_{i=1}^{n_j}\frac{1}{V}\int_V\delta(\mathbf{r}'-\mathbf{r}_{ij}^{\Gamma})\,{\rm d}\mathbf{r}_{ij}^{\Gamma}=\begin{cases}\frac{3v_j}{8\pi R_j^3}, & z'=0,\\ \frac{3v_j}{4\pi R_j^3}, & z'>0.\end{cases} \tag{20}$$

Then taking into account (19) and (20) and definition of an improper integral we obtain

$$\langle I_2\rangle_{\rm conf}=\sum_{j=1}^{N}(D_m-D_j)\frac{3v_j}{4\pi R_j^3}\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\frac{1}{2}G\frac{\partial c_0}{\partial z'}\bigg|_{z'=0}+\int_{+0}^{\infty}G\frac{\partial c_0}{\partial z'}\,{\rm d}z'\right\}{\rm d}x'\,{\rm d}y'\,{\rm d}t'. \tag{21}$$

Because of $G|_{z'=0}=0$ and expressions (18) and (21) take place, in consequence we obtain an expression for calculating the approximate admixture concentration field averaged over the ensemble of phase configurations in the multiphase semispace with spherical inclusions in the form

$$\langle c(\mathbf{r},t)\rangle_{\rm conf}=c_0(z,t)+\sum_{j=1}^{N}v_j\int_0^t\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\int_0^{2R_j}G\left(\frac{z'}{R_j}-1\right)^3\right.\times\left[\rho^*\frac{\partial c_0}{\partial t'}-D_j^*\frac{\partial^2 c_0}{\partial z'^2}\right]{\rm d}z'+\int_{2R_j}^{\infty}G\left[\rho^*\frac{\partial c_0}{\partial t'}-D_j^*\frac{\partial^2 c_0}{\partial z'^2}\right]{\rm d}z'+\left.\frac{3(D_m-D_j)}{4\pi R_j^3}\int_{+0}^{\infty}G\frac{\partial c_0}{\partial z'}\,{\rm d}z'\right){\rm d}x'\,{\rm d}y'\,{\rm d}t'. \tag{22}$$

The averaged field of admixture concentration in a semispace with nonhomogeneous structure we find substituting the corresponding expressions for Green function (15) and the admixture concentration in homogeneous medium with characteristics of the basic phase (11) into (22). And we also allow for that $\bar{\rho}_0\equiv 1$. As a result we obtain

$$\frac{1}{c^*}\langle c(\mathbf{r},t)\rangle_{\rm conf}=\sum_{j=1}^{N}\left[\left(1+v_j\frac{3(D_m-D_j)\pi^2 z}{16R_j^3D_0}\right){\rm efrc}\left\{\frac{z}{2\sqrt{D_0t}}\right\}+v_j\frac{\tilde{\rho}\pi^2 z}{2t}\sqrt{\frac{\pi}{D_0}}+v_j\frac{\pi^2\tilde{\rho}}{4t}\left\{\int_0^t\frac{1}{\sqrt{t'}}A_2(z,t)\times\exp\left\{-\frac{z^2}{4D_0(t-t')}\right\}{\rm d}t'+\exp\left\{-\frac{z^2}{4D_0t}\right\}\times\int_0^t\left[\exp\left\{-\frac{1}{t-t'}\left[R_j^2\frac{t}{t'}+\frac{zt'}{4D_0t}\right]\right\}\times\left(A_1(z,t)\sinh\left\{\frac{R_jz}{D_0(t-t')}\right\}-A_3(z,t)\cosh\left\{\frac{R_jz}{D_0(t-t')}\right\}\right)+A_4(z,t){\rm erf}\left\{a(t)\frac{z}{2}\right\}-A_+{\rm erf}\left\{a(t)\left[R_j\frac{t}{t'}+\frac{z}{2}\right]\right\}+A_-{\rm erf}\left\{a(t)\left[R_j\frac{t}{t'}-\frac{z}{2}\right]\right\}\right]{\rm d}t'\right\}, \tag{23}$$

where

$$\tilde{\rho}=\rho^*-\frac{D_j^*}{D_0},\quad a(t)=\sqrt{\frac{t'}{D_0t(t-t')}},\quad A_1(z,t)=\left(\frac{zt}{R_jt'}\right)^2,$$

$$A_2(z,t)=2\sqrt{A_1}\left[A_1+10\frac{b_2^2}{R_j^2}+3\right],\quad A_3(z,t)=\sqrt{A_1}\left[A_1+10\frac{b_2^2}{R_j^2}+1\right],$$

$$A_4(z,t)=\sqrt{\pi}\frac{2b_1}{R_j}\left[6+\frac{3}{2}b_1+\frac{2b_2^2}{R_j^2}\left(3+b_1\left[3+\frac{1}{4}b_1\right]\right)\right],$$

$$A_{\pm}(z,t)=\sqrt{\pi}\left[\frac{b_2^3}{R_j^3}b_1^{3/2}\pm\frac{3}{R_j^2}b_2^2b_1+b_3\pm 1\right]+\frac{3b_2}{R_j}\left[1+(1+b_1)\frac{2b_2^2}{R_j^2}\pm b_3\right];$$

$$b_1=\frac{z^2t'}{D_0t(t-t')},\quad b_2^2=\frac{D_0(t-t')t'}{t},\quad b_3=\frac{3b_2}{R_j}\sqrt{b_1}.$$

5. Results of numerical calculations

Figs. 2–5 illustrate the influence of nonhomogeneities of material structure on distributions of the averaged

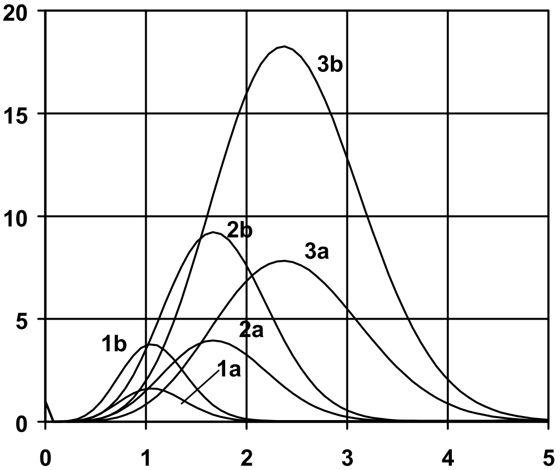

Fig. 2. The averaged admixture concentration in different dimensionless instants.

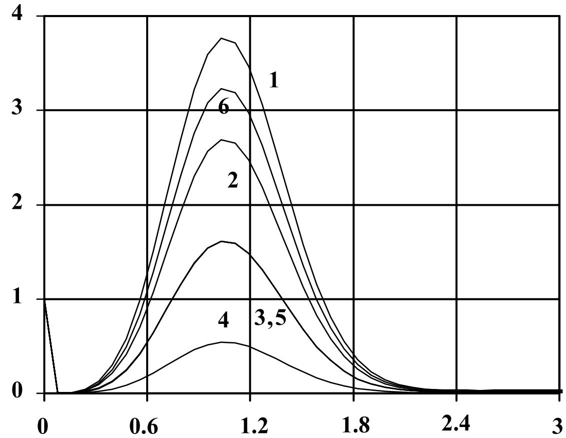

Fig. 3. Dependence of the averaged concentration on the reduced diffusion coefficient $\overline{D}_1$.

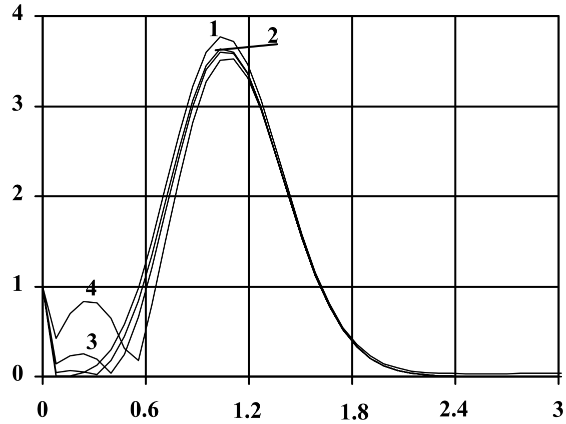

Fig. 4. Influence of the characteristic radius of spherical inclusions on the averaged admixture concentration.

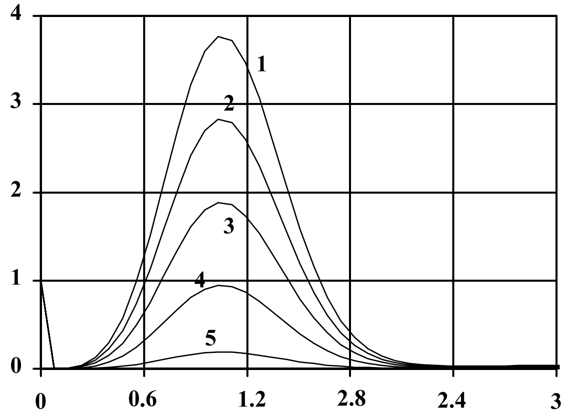

Fig. 5. The averaged admixture concentration for different values of the inclusion volumetric fraction.

admixture concentration in a semispace under action of a constant mass source on the body boundary as an example of a two-phase body. Numerical calculations have been done in the dimensionless variables $\xi = z/z_0$ ($z_0 = 1$ m) and $Fo = D_0 t/z_0^2$ [2]. It is taken $\overline{D}_1 = D_1/D_0 = 0.5$, $\bar{\rho}_1 = 1.2$, $\overline{R}_1 = R_1/z_0 = 0.1$, $Fo = 0.1$, $v_1 = 0.2$. The dimensionless coordinate ξ has been laid off as abscissa, ratio of the averaged admixture concentration to its value on the body boundary c^* has been laid off as ordinate. Concentration distributions are presented for different values of Fourier number $Fo = 0.1$, 0.25, 0.5, curves 1–3, respectively, in Fig. 2. Here curves **a** are given for the reduced diffusion coefficient $\overline{D}_1 = 1.5$, curves **b** correspond the value $\overline{D}_1 = 0.5$. Fig. 3 demonstrates the influence of the reduced diffusion coefficient value on the distribution of the averaged concentration field; here $\overline{D}_1 = 0.5$, 0.7, 0.9, 1.1, 1.5, 1.8, curves 1–6, respectively. Dependence of the admixture particle concentration on characteristic radius of spherical inclusions is shown in Fig. 4; here $\overline{R}_1 = 0.1$, 0.01, 0.003, 0.001, curves 1–4, respectively. Fig. 5 illustrates the behaviour of concentration field depending on the volumetric fraction of inclusions. Here curves 1–5 correspond values $\langle c(\xi, Fo)\rangle / c^*$ under $v_1 = 0.2$, 0.15, 0.1, 0.05, 0.01.

Performed analysis of the obtained results shows that distinctions in diffusive properties of randomly distributed phases can cause essential change of character of the admixture concentration field in the body. At that in quantitative description of mass transfer in such bodies it is necessary to allow for explicitly both different values of the admixture diffusion coefficient and its jump discontinuities at interphase boundaries. Numerical calculations of the function under consideration show increase of admixture concentration in subsurface domain of the body with spherical inclusions (see Figs. 2–5). We suppose that presence of inclusions with distinct diffusive properties gives rise to concentration gain because of significant quantity of admixture particles can be accumulated in a vicinity of interphase surfaces.

Let us note that the maximum value of the admixture concentration increases and moves into body depth with time (see Fig. 2). Changes in material characteristics affect essentially on behaviour and values of the averaged function of concentration in a nonhomogeneous medium. For example, decreasing characteristic sphere radius at the same volumetric fraction of inclusions (i.e. quantity of spherical inclusions grows) causes appearance of the second maximum of the averaged concentration field in subsurface body domain (see Fig. 4). Note that increase of difference between admixture diffusion coefficients in the matrix and inclusions can cause increase of the averaged concentration in the body (see Fig. 3). Enlarging volumetric fraction of spherical inclusions causes the same effect (see Fig. 5).

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